Test exercises for the waiting list

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Deadline: 13 July 2016

Problem 1:
Prove or refute:

(i) If a prime is divided by 30, the remainder is 1 or a prime.
(ii) If a prime is divided by 60, the remainder is 1 or a prime.

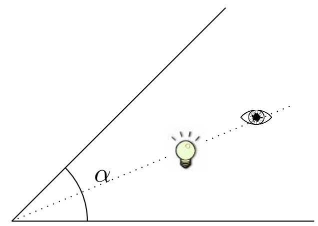
Problem 2:
From a three-letter alphabet all possible words are constructed. During construction, certain combinations of consecutive letters are considered forbidden. It is known that each forbidden combination has length of at least two, and that all forbidden combinations are of different lengths. Do arbitrarily long allowed words exist?

Problem 3:
A seminar has n participants. Some of them are acquainted with each other; each pair of students who don't know each other has exactly two common acquaintances, and each pair of acquainted students have no common acquaintances. Show that each participant is acquainted with the same number of participants.

Problem 4:
Prove that a natural number $p \geq 2$ is prime if and only if it divides $(p-1)! + 1$.

Problem 5:
Is there a nonconstant polynomial in $\mathbb{Z}[x]$ whose value for each natural argument is prime?

Problem 6:
Assume that two mirrors form a sharp angle α, that these mirrors are infinitely long in one direction, and that somewhere along the bisectrix a light bulb and an observer are placed:



Neglecting the sizes of the light bulb and of the observer, how many bulb images does the observer see?